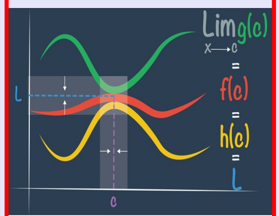
Calculus I

Lecture 26



Feb 19-8:47 AM

Class QZ 23

Find fave for $f(x) = \sqrt[3]{x}$ on $[1, 8]$

$$\boxed{f_{ave} = \frac{1}{b-a}\int_a^b f(x)\,dx}$$

$$f_{ave} = \frac{1}{8-1}\int_1^8 \sqrt[3]{x}\,dx$$

$$= \frac{1}{7}\int_1^8 x^{1/3}\,dx$$

$$= \frac{1}{7}\cdot \frac{x^{4/3}}{4/3}\Big|_1^8$$

$$= \frac{3}{28}\left[8^{4/3} - 1^{4/3}\right]$$

$$= \frac{3}{28}\left[(2^3)^{4/3} - 1\right] = \frac{3}{28}\left[16-1\right]$$

$$= \frac{3}{28}\cdot 15$$

$$= \boxed{\frac{45}{28}} \checkmark$$

May 19-11:09 AM

Take Home class Quiz 24

Find f_{ave} for

1) $f(x) = \frac{x}{\sqrt{3+x^2}}$ on $[1,3]$

$f_{ave} = \frac{1}{3-1}\int_1^3 \frac{x}{\sqrt{3+x^2}}\,dx$

$u = 3+x^2$
$du = 2x\,dx$
$\frac{du}{2} = x\,dx$
$x=1 \to u=4$
$x=3 \to u=12$

$= \frac{1}{2}\int_4^{12} \frac{1}{\sqrt{u}}\frac{du}{2}$

$= \frac{1}{4}\int_4^{12} u^{-1/2}\,du$

$= \frac{1}{4}\cdot \frac{u^{1/2}}{1/2}\Big|_4^{12}$

$= \frac{1}{2}\left[\sqrt{u}\Big|_4^{12}\right]$

$= \frac{1}{2}\left[\sqrt{12}-\sqrt{4}\right] = \frac{1}{2}\left[2\sqrt{3}-2\right] = \boxed{\sqrt{3}-1}$

2) $f(x) = \cos^4 x \sin x$ on $[0,\pi]$

$f_{ave} = \frac{1}{\pi - 0}\int_0^{\pi}\cos^4 x \sin x\,dx$

$= \frac{1}{\pi}\int_0^{\pi}\cos^4 x \sin x\,dx$

$u = \cos x$
$du = -\sin x\,dx$
$-du = \sin x\,dx$

$= \frac{1}{\pi}\int_1^{-1} u^4(-du)$

$= -\frac{1}{\pi}\int_1^{-1} u^4\,du$

$= -\frac{1}{\pi}\cdot -\int_{-1}^{1} u^4\,du = \frac{1}{\pi}\frac{u^5}{5}\Big|_{-1}^{1}$

$= \frac{1}{5\pi}\left[1^5 - (-1)^5\right] = \frac{1}{5\pi}[1+1]$

$= \boxed{\frac{2}{5\pi}}$

May 19-11:03 AM

Suppose $f(x) = \int_{u(x)}^{v(x)} g(t)\,dt$ and $u(x)$ & $v(x)$

are both differentiable,

$$\boxed{f'(x) = g(v(x))\cdot v'(x) - g(u(x))\cdot u'(x)}$$

ex $f(x) = \int_1^{x^3} \sin t^2\,dt$

1) Find $f(1)$

$f(1) = \int_1^{1^3} \sin t^2\,dt = \int_1^1 \sin t^2\,dt = \boxed{0}$

2) Find $f'(x)$

$f'(x) = \sin(x^3)^2\cdot\frac{d}{dx}[x^3] - \sin 1^2\cdot\cancel{\frac{d}{dx}[1]}^{0}$

$= \sin x^6 \cdot 3x^2 = \boxed{3x^2 \sin x^6}$

May 21-9:05 AM

$$f(x) = \int_{x^2}^{x^3} \frac{t^2-1}{t^2+1}\,dt$$

1) $f(1) = \int_{1^2}^{1^3} \frac{t^2-1}{t^2+1}\,dt = \int_{1}^{1} \frac{t^2-1}{t^2+1}\,dt = 0$

2) $f'(x) = \frac{(x^3)^2-1}{(x^3)^2+1}\cdot 3x^2 - \frac{(x^2)^2-1}{(x^2)^2+1}\cdot 2x$

$$= \frac{x^6-1}{x^6+1}\cdot 3x^2 - \frac{x^4-1}{x^4+1}\cdot 2x$$

May 21-9:12 AM

Given $f(x) = \int_{\tan x}^{x^2} \frac{1}{\sqrt{2+t^4}}\,dt$

1) find $f(0)$

$$f(0) = \int_{\cancel{\tan 0} \to 0}^{\cancel{0^2} \to 0} \frac{1}{\sqrt{2+t^4}}\,dt = \boxed{0}$$

2) find $f'(x)$

$$f'(x) = \frac{1}{\sqrt{2+(x^2)^4}}\cdot 2x - \frac{1}{\sqrt{2+\tan^4 x}}\cdot \sec^2 x$$

May 21-9:16 AM

on what interval $f(x) = \int_0^x \frac{t^2}{t^2+t+2}\,dt$ is

Concave down?

$f''(x) < 0$

$f'(x) = \frac{x^2}{x^2+x+2} \cdot 1 - \frac{0^2}{0^2+0+2} \cdot 0$

$f'(x) = \frac{x^2}{x^2+x+2}$ $\quad f' = 0 \to x = 0$, f' is never und.

$f''(x) = \frac{2x(x^2+x+2) - x^2(2x+1)}{(x^2+x+2)^2}$

$= \frac{2x^3+2x^2+4x-2x^3-x^2}{(x^2+x+2)^2}$

$= \frac{x^2+4x}{(x^2+x+2)^2}$ $\quad x(x+4)$

$x^2+4x=0 \to x=0,\ x=-4$

x		-4		0	
$f'(x)$	+		+	•	+
$f''(x)$	+	•	—	•	+
$f(x)$	CU		CD		CU

$(-4, 0)$

Due Wednesday in class

1) $f(x) = \int_{\sqrt{x}}^{x^3} \cos t^2\,dt$, find $f(1)$ & $f'(1)$

2) $f(x) = \int_0^x (1-t^2)\cos^2 t\,dt$, on what interval is $f(x)$ increasing?